J. Fluid Mech. (1978), *vol.* 89, *part* 3, *pp.* 589–606
Printed in Great Britain

Turbulence memory in self-preserving wakes

By PAUL M. BEVILAQUA† AND PAUL S. LYKOUDIS‡

School of Aeronautics, Astronautics and Engineering Sciences,
Purdue University, West Lafayette, Indiana 47907

(Received 21 August 1974 and in revised form 29 June 1978)

The persistence of the large vortices formed at the origin of wakes and mixing layers constitutes a kind of memory of initial conditions by the turbulence. In order to study the fading of this turbulence memory, and its effect on the rate of approach to the fully developed state, two wakes with different initial conditions have been examined experimentally. The wake of a sphere was compared with the wake of a porous disk which had the same drag, but did not exhibit vortex shedding. Measurements were made of the mean and fluctuating velocities, the anisotropy of the turbulence, and the intermittency. It was found that the wake of the sphere developed self-preserving behaviour more rapidly than the wake of the disk, and that even after both wakes became self-preserving there were differences between them in the structure of the turbulence and the scale of the mean flow. From this it is concluded that the behaviour of self-preserving wakes does not depend on the drag alone, but also on the structure of the dominant eddies. Generalizing these results, it is suggested that reported differences in the value of the entrainment constant of jets, wakes, and mixing layers are due to differences in the structure of the dominant eddies, rather than differences in the type of flow.

1. Introduction

In a series of monographs, Townsend (1956, 1970, 1976) developed the hypothesis that the structure of the turbulence in all self-preserving flows is the same. The dominant eddies resemble a vortex pair inclined along the principal axis of the mean strain rate. According to the hypothesis, these eddies originate in the stretching of initially unorganized turbulence by the mean shear flow; the development of the flow becomes self-preserving if the dominant eddies attain a condition of moving equilibrium, which is a state of balance between amplification by the mean strain and dissipation by interaction with the smaller eddies. Because this fully developed state depends only on the dynamics of the turbulent energy balance, the equilibrium hypothesis implies that the structure of self-preserving flows shows no memory of initial conditions; development of the flow depends on the net force on the fluid, but not the details of how the force was applied. Variations in the rate of decay between flows of different 'type'§ are attributed to differences in the degree of straining. Thus, according to

† Present address: Rockwell International, Columbus Aircraft Division, Columbus, Ohio 43216.

‡ Present address: School of Nuclear Engineering, Purdue University, West Lafayette, Indiana 47907.

§ Here we use 'type' to mean both the kind of flow (for example, jet or wake) and the geometry of the flow (for example, round jet or plane jet).

Townsend's hypothesis, a flow becomes self-preserving because the turbulence attains a characteristic equilibrium state which depends only on the flow 'type' and the momentum integral.

However, in recent experimental work, there is considerable evidence that a memory of the initial conditions persists into the region of self-preservation. Papailiou (1971) and Papailiou & Lykoudis (1974) discovered that the vortices shed into the wake of a cylinder remain coherent at higher Reynolds numbers and for greater distances downstream than previously suspected, while Bevilaqua & Lykoudis (1971) observed similar vortices in the wake of a sphere. At about the same time, Brown & Roshko (1971) independently found that large-scale two-dimensional vortices also occur in the turbulent mixing layer. The persistence of the vortices shed into these flows at their origin constitutes a memory of initial conditions by the turbulence. The existence of these vortices in the region of self-preservation is not consistent with the equilibrium hypothesis that shear flows become self-preserving as the memory of initial conditions fades.

The purpose of the present study was to understand the relationship of these transverse vortices to the self-preserving development of turbulent shear flows. To this end, the initial evolution of two turbulent wakes of the same 'type' has been examined experimentally. The wake of a sphere has been compared with the wake of a porous disk, with approximately the same drag, but which did not develop a recirculation region or exhibit vortex shedding. It was observed that, owing to the slow fading of the memory of initial conditions, there were differences in the large eddy structure of the far wake and, even though the drag of both models was the same, two distinct self-preserving states evolved.

In §2 the experimental apparatus used in this study is described, and in §3 the experimental results are presented. The meaning of these results is discussed in §4, and it is suggested that a distinction must be made between fully developed, or equilibrium, flows and flows that are merely self-preserving.

2. Experimental apparatus

In order to remove the possibility that the vortex structure was determined by end effects, we decided to study round, rather than plane wakes. A wind tunnel with low turbulence levels in the main stream was designed especially for this study. It is of the open return type. Room air is drawn through a 20:1 contraction cone, the test section, and a short diffuser into a plenum box. This plenum contains two centrifugal blowers which exhaust the flow back to the atmosphere. The test section consists of three transparent Plexiglas cylinders, each 120 cm long and having a circular cross-section 45 cm in diameter. The contraction cone has a circular cross-section also. Before entering the contraction, the flow passes through a sheer nylon dust filter and six layers of fine mesh screen. These factors combine to produce a uniform flow and low turbulence levels in the test section. The turbulent fluctuations are less than 0·3 % of the mean velocity and the average velocity at each point in the field of measurement varies less than 1 % from the centre-line value. The static pressure gradient through the test section is approximately 0·5 mm of Hg per metre.

A commercially manufactured ball-bearing 2·54 cm in diameter served as the test sphere. It was suspended on the tunnel centre-line with two 0·5 mm piano wires passed through holes drilled in the model. The screen disk had a diameter of 2·54 cm also. It was constructed by removing two strands from perpendicular diameters of a circular piece of 18 mesh screen and weaving the support wires through in their place. The porosity of the screen, the ratio of open area to total area being 0·82, was chosen by integrating the velocity profiles in the wakes of a series of rings and disks of varying porosity. We found that a disk of 18 mesh screen had approximately the same drag as the sphere. During this investigation, we observed that unless the diameter of the hole in a ring was at least half the total diameter its wake was virtually indistinguishable from that generated by a solid disk of equal size. Apparently, the resistance of a small hole is such that a ring obstructs the flow almost as if it were solid.

The tunnel blockage ratio of these models was 0·003. Achenbach (1974) established that there are no blockage effects at this scale in a tunnel with a circular cross-section. No vibrations of either test body could be detected with an alignment telemicroscope, and rotation of the test section containing the models through 90°, 180° and 270° verified that the wake was axisymmetric.

A Thermo-Systems linearized anemometer was used to determine the mean and fluctuating components of the velocity. The sensing elements were platinum hot films, 25 μm in diameter, also manufactured by TSI. Measurement of the streamwise velocity components was made with a single horizontal sensor. Other components were derived using analog circuitry and the output of a dual sensor X-probe. In the present study, the field of measurement was limited to the vertical plane passing through the axis of the tunnel and the midpoint of the models; the wake of the support wires formed an X which did not intersect this plane.

Since there is a measure of subjectivity involved in defining the instant at which a probe signal becomes turbulent, there are almost as many discrimination schemes as there are investigators studying intermittency. In the present work the intermittency function, which has the value unity in the turbulence and zero outside it, was generated with an analog circuit, according to the following scheme. The presence of turbulence was detected with a probe having two parallel hot-film sensors 2·5 mm apart. At this separation, the small-scale turbulent fluctuations were uncorrelated, while the larger-scale irrotational fluctuations were well correlated. Differencing the output of the two sensors thus generated a significantly larger signal during the turbulent intervals. Kibens (1968) has pointed out that this signal is proportional to the turbulent vorticity fluctuations, since $\omega \sim \Delta u/\Delta y$ in wake flows. This detector probe output was then differentiated with respect to time and rectified, after Townsend (1949), in order to emphasize the high frequency character of the turbulence. The amplitude of the resulting signal was examined in a discriminator stage, where it was compared with a reference level set just slightly above the noise level in the non-turbulent periods. The output of the discriminator is a random square wave, the intermittency function, which is unity when the test signal is greater than the reference, and zero otherwise. The intermittency factor γ was determined by summing the intermittency function with an ITRON counter-timer. This function was also used with a VIDAR integrating digital voltmeter to evaluate the turbulent zone average of the Reynolds stresses. A circuit diagram and details of the entire procedure are given in Bevilaqua (1973).

A Reynolds number of 10000 was chosen for this study, since we felt it would be generally agreed that although the wake is fully turbulent, vortex shedding is an important part of the formation process. Further, this choice permits comparison with several earlier studies of the low speed wake performed in the Reynolds number range from 8000 to 12500.

3. Experimental results

3.1. *Initial conditions and early development*

At the Reynolds number chosen for this study, the boundary layer on the front face of the sphere is laminar. Transition occurs just downstream of the separation point in the resulting free shear layer. Entrainment by this shear layer from the region behind the sphere produces a reverse flow on the wake axis that results in the formation of a toroidal eddy or ring vortex in the near wake. Our measurements show that at low Reynolds numbers this vortex is confined to the shear layer, but that at higher Reynolds numbers it broadens to include the entire wake bubble. The radial distribution of the mean velocity in figure 1 shows the development of the ring vortex as the free-stream velocity is increased. Further increases in the velocity to $Re = 30000$ produced only small additional changes in the profile.

The vorticity shed with the boundary layer accumulates in this vortex and is periodically shed into the wake. The frequency of vortex shedding and the structure of the large eddies were not determined in this study. However, previous visualization experiments (Bevilaqua & Lykoudis 1971) revealed the presence of transverse vortices in the developed wake. More recently, Achenbach (1974) concluded that the separation point rotates around the sphere at the Strouhal frequency $fd/u = 0{\cdot}15$ at $Re = 10000$ while Pao & Kao (1977) suggested that the vorticity is shed in a series of vortex loops which are linked to form a double helix structure.

We found that the flow passing through the porous disk causes significant differences in the character of its wake. Castro (1971) observed that no ring vortex develops behind perforated plates when the porosity of the plate is 0·3 or greater. Although a disk was used in the present study, its porosity was well above this critical value and a similar effect was seen. Within the first diameter downstream of the model, the individual wake of each wire in the mesh could be detected. As these wakes merged, the mean velocity profile slowly developed the shape characteristic of round wakes. Apparently, enough air flows through the disk to prevent the development of a recirculating eddy.

The characteristic profiles take considerably longer to evolve in the wake of the disk than they do behind the sphere. This can be seen in figure 2, which shows the axial variation of the mean and fluctuating velocity profiles in the near wake of both models. In the wake of the disk, the combined effects of the stagnation pressure ahead of the model and the entrainment behind it produce a velocity peak on the axis, while the velocity peak in the wake of the sphere is smaller and associated with the reverse flow due to the ring vortex. Owing to the large mean rate of strain, the turbulent fluctuations in the shear layer at the boundary of both wakes are initially almost twice as large as those in the central core. As the turbulence diffuses, these shear layers thicken and merge. Several diameters further downstream, the profiles appear to attain their characteristic, self-preserving form. This near wake region is only about five diameters long behind the sphere, but in the wake of the disk, it is almost twenty diameters long.

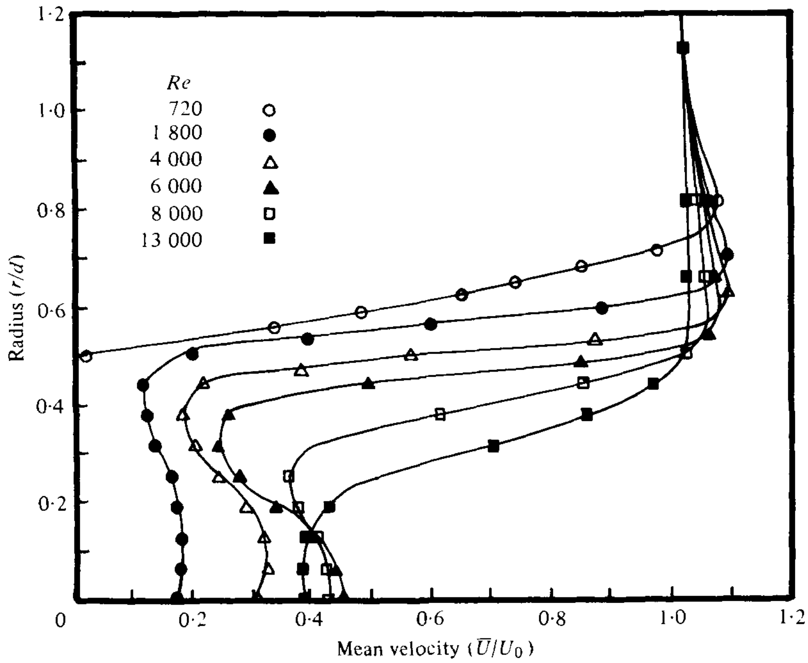


FIGURE 1. Ring vortex behind the sphere; $x/d = 0{\cdot}8$.

The absence of vortex shedding from the porous disk may be inferred from these results, and also by comparison with Castro's work. However, more direct evidence is contained in our measurements of the intermittency profiles. Figure 3 compares the diffusion of the wakes in terms of the intermittency factor. The fully turbulent core is confined to the first fifteen diameters behind the sphere. Beyond that distance the corrugation amplitude of the turbulent interface has increased to a size of the order of the wake radius, so that periods of irrotational flow occur on the axis. Although the centre-line value of the intermittency factor decreases downstream of the sphere, the variation is slight and no systematic decay was observed within the first 120 diameters considered in this study. Similar behaviour is seen in the data of Freymuth & Uberoi (1973) for the near wake of a sphere at $Re = 19000$ and in the measurements of Oswald & Kibens (1971) for the wake of a solid disk at $Re = 12500$. At greater distances downstream Baldwin & Sandborn (1968) found in the wake of a disk that the centre-line intermittency decays linearly, while Riddhagni, Bevilaqua & Lykoudis (1971) observed that the intermittency decays as $(x/d)^{-0{\cdot}15}$ in the far wake of a sphere for $Re = 78000$. These differences may be related to variations in the free-stream turbulence intensity, or to more fundamental differences in the structure of the wake.

On the other hand, intermittency was not observed until approximately twenty diameters downstream of the porous disk. In the near wake, the turbulent fluctuations diminish abruptly, but continuously across the wake boundary. The boundary itself is initially smooth and uncontorted. The appearance of intermittency beyond twenty diameters was preceded by random bursts of high frequency turbulence upstream in the shear layer. The profiles shown at $x/d = 10$ in the wake of the disk give the intermittency of these high frequency bursts. The boundary was uncontorted and the

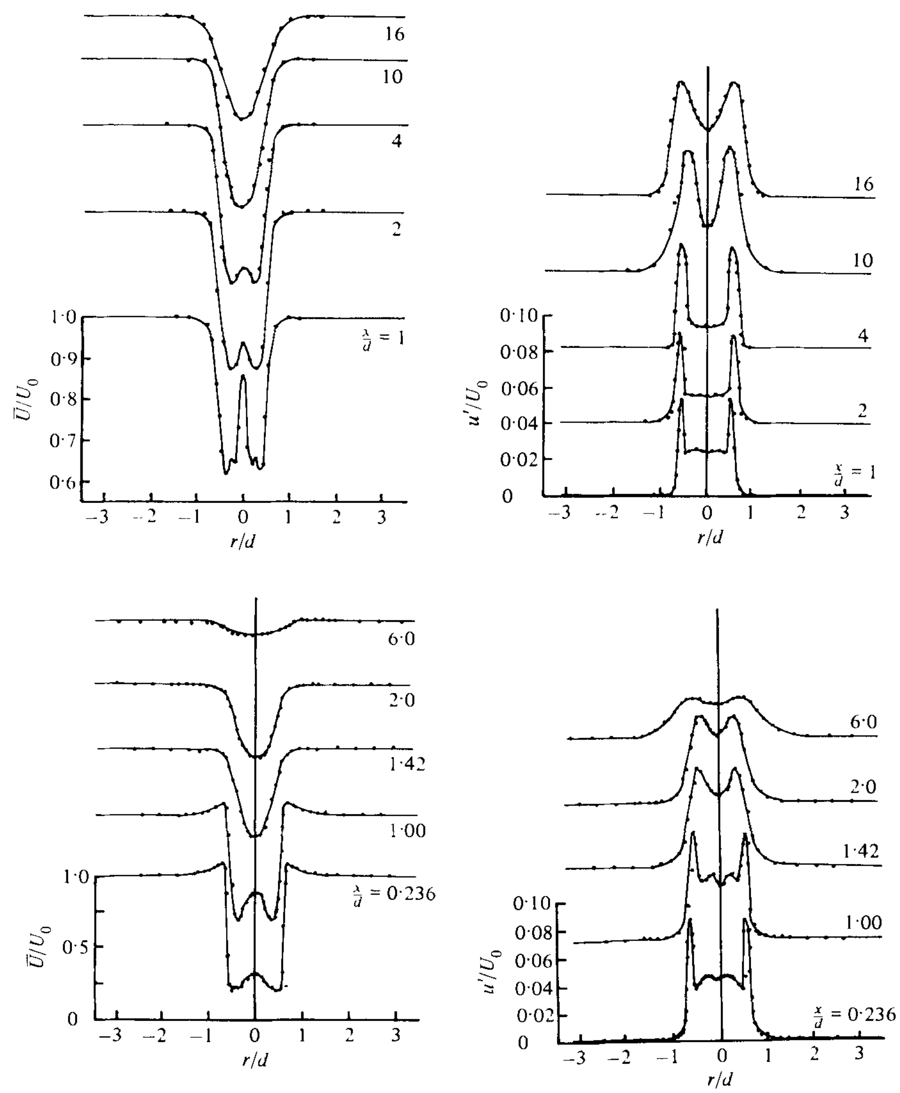


FIGURE 2. Mean velocity (left) and turbulence intensity (right) in the near wakes of the porous disk (above) and sphere (below).

wake was fully turbulent at this point. The bursts most likely correspond to the onset of an instability of the shear layers.

From the measurements of intermittency shown in figure 3, it can be seen that the amplitude of the surface waves remains relatively smaller in the wake of the disk than in the wake of the sphere. The fully turbulent core extends more than one hundred diameters downstream of the disk. Although the radial variation of the intermittency factors does not develop a self-preserving form behind either model within the range investigated, the fluctuations of the interface have a Gaussian distribution about the mean boundary position. The absence of intermittency in the near wake of the disk and the relatively smaller amplitude of the surface waves that do develop are indications that large eddies, which cause the motion of the interface, are not shed from the disk.

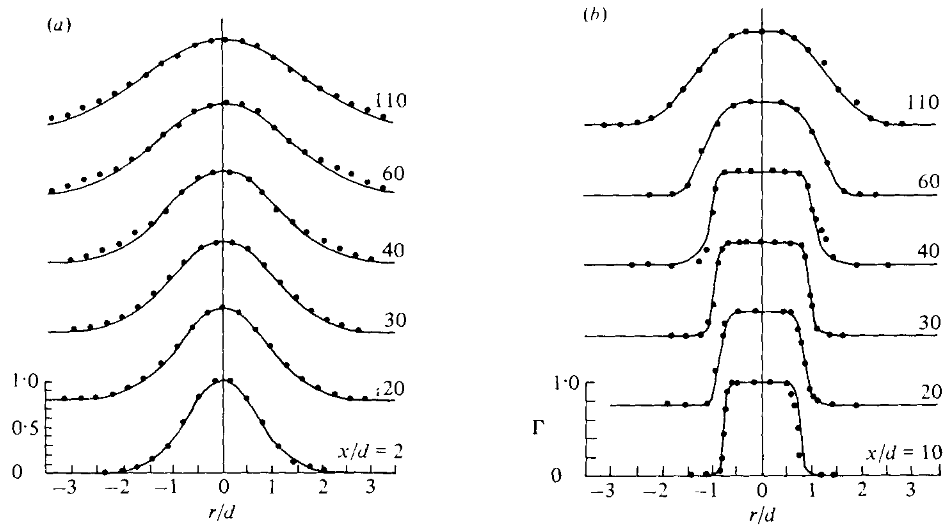


FIGURE 3. Intermittency factor profile in the wakes of the sphere (a) and porous disk (b).

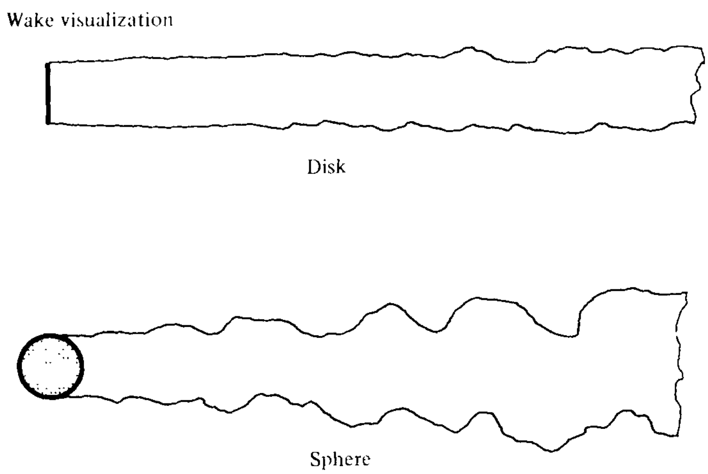


FIGURE 4. Wake visualization for a sphere and a porous disk in a water tow tank at $Re = 10000$.

These measurements suggested differences in the wake structure that could be seen if the flow were visualized. In order to make these differences apparent, scaled-up models of the sphere and disk were towed through a water-filled trough. Dye was injected into the wake of the sphere at the rear stagnation point, and every 45° around the perimeter of the screen disk through holes drilled in a hoop of brass tubing. Figure 4 shows tracings made from motion pictures taken during the visualization of each wake. The diameter of the porous disk was almost three times that of the sphere, but the models are shown to the same scale because the Reynolds number was the same in each case, $Re = 10000$. No measurements of drag were made.

The wake of the sphere exhibits the characteristic 'snaking' behaviour which leads to intermittency on the wake centre-line. This phenomenon was first reported by Hwang & Baldwin (1966). The eventual appearance of waves on the surface of the

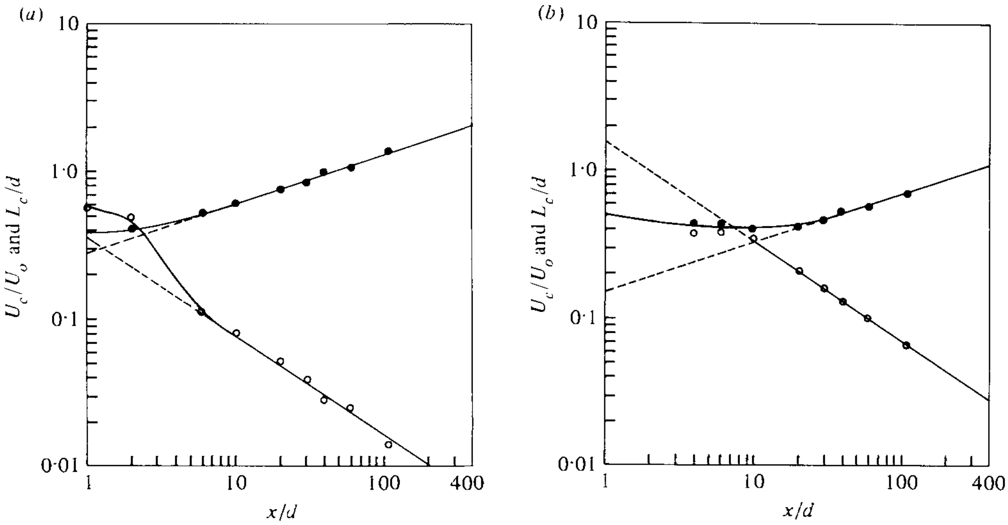


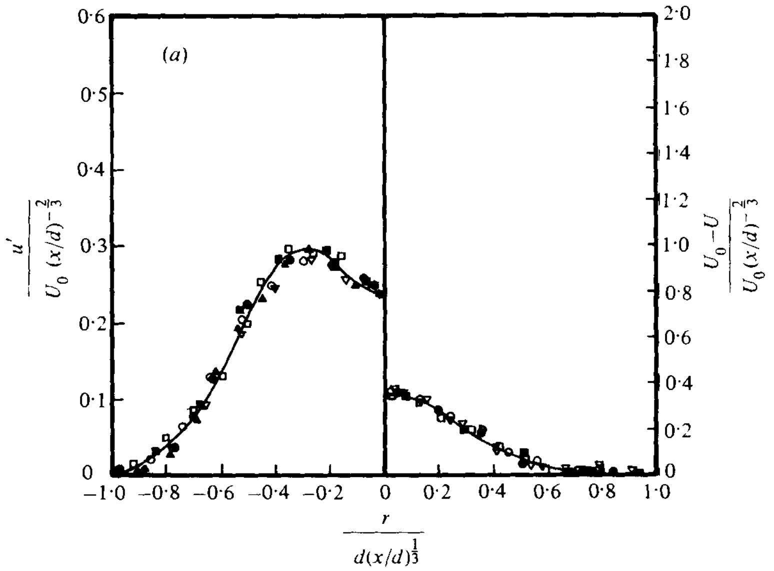
FIGURE 5. Development of the characteristic length and velocity scales in the wakes of the sphere (a) and porous disk (b). ○, centre-line velocity defect; ●, radius of the wake at which $U/U_c = e^{-\frac{1}{2}}$.

disk wake suggests that there is an active instability of the shear layer, but the absence of the snaking motion is significant. It has been suggested that the snaking may be characteristic of the instability of an axisymmetric flow; however, on the basis of the present results, we believe that snaking is associated with the mode of vortex shedding. The structure seen in these sketches is in agreement with the measured distribution of the intermittency factor.

3.2. *Self-preserving development*

The mean velocity and Reynolds stress profiles are geometrically similar in the region of self-preservation, and may be normalized by characteristic scales of length and velocity. A convenient velocity scale U_c is the centre-line defect velocity; the length scale L_c is defined as the radial co-ordinate at which $U/U_c = \exp(-\frac{1}{2})$. In figure 5 the variation of U_c and L_c downstream of each model is shown for comparison. The development of these scales becomes self-preserving within ten diameters of the sphere and within twenty diameters of the porous disk. However, it can be seen that a different self-preserving state has evolved in each case. This difference in the scale and structure of the two wakes is also apparent in the velocity profiles. In figure 6 our measurements of the self-preserving mean velocity defect and turbulence intensity profiles are plotted with the same normalizing parameters, $U_s = U_0(x/d)^{-\frac{2}{3}}$ and $L_s = d(x/d)^{\frac{1}{3}}$, to emphasize the difference in the scale of these two wakes. The intensity of the turbulence relative to the defect velocity is greater in the wake of the sphere, which is consistent with the more rapid diffusion of the sphere wake seen in figures 3 and 5.

Relatively little has been published on the development of self-preserving behaviour in round wakes. However, available data for the wake of the sphere suggests that the virtual origin of the wake is very near the actual position of the model. Riddhagni *et al.* (1971) found $x_0 = 0$ for $Re = 78000$ while Freymuth & Uberoi (1973) found the



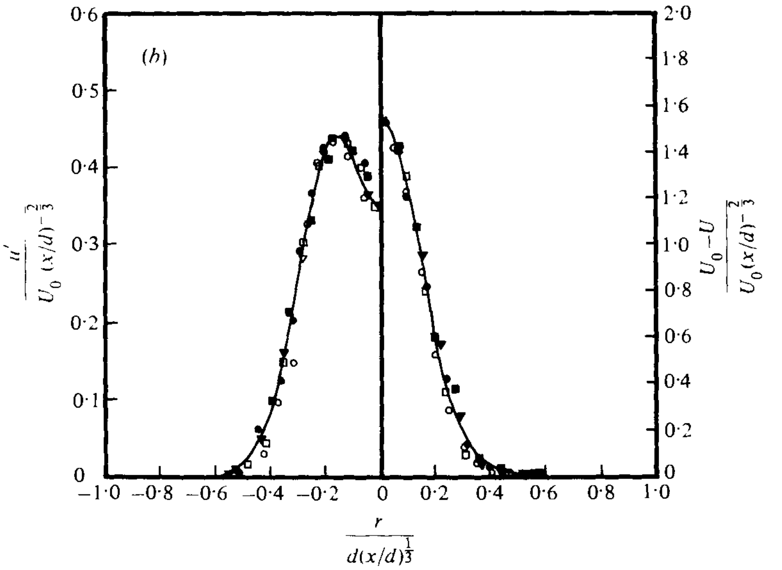


FIGURE 6. Self-preserving forms of the mean and fluctuating velocities in the wakes of the sphere (*a*) and porous disk (*b*). Non-dimensional distance downstream, x/d: ○, 20; □, 30; ▼, 40; ●, 60; ■, 110.

virtual origin of the temperature wake to coincide with the sphere centre for $Re = 4300$. For $Re = 8600$ Uberoi & Freymuth (1970) reported the virtual origin of the momentum wake to be at $x_0 = 12d$, but correcting their centre-line decay data for intermittency would move the virtual origin to the position of the sphere in this case also. Because the structure of the wake develops gradually over some distance, the point at which the flow actually becomes self-preserving cannot be defined. In general, the mean velocity profiles become self-preserving first, followed by the Reynolds stresses, and then the higher-order turbulence moments. The data of Gibson, Chen & Lin (1968) for $Re = 65000$ suggest that the decay of a scalar very quickly becomes self-preserving, before $x/d \sim 10$. This is consistent with the results of Riddhagni *et al.* (1971), which exhibit self-preservation by $x/d \sim 8$ for $Re = 78000$. There is even less data available for the development of self-preservation in disk wakes, but Carmody (1963) identified a region of self-preservation for $x/d > 15$.

According to the equilibrium hypothesis, straining of the turbulence by the mean shear flow intensifies the turbulent vortex filaments aligned with the principal axis of strain and leads to the appearance of the vortex pair eddies. Thus, the evolution of the equilibrium structure is indicated by the anisotropy of the turbulent velocity fluctuations aligned with the principal axes of the mean strain rate; that is, at $\pm 45°$ to the direction of flow. In reality the direction of the vorticity maximum may not be the same as the principal strain axis, because the vortex lines are also being rotated by the mean shear flow. However, this does not invalidate the basic argument. It is only necessary that the preferred direction be close to 45°, and the experimental evidence (Grant 1958; Kline *et al.* 1967) indicates that this is the case.

We have measured the turbulence intensity in both wakes at $\pm 45°$ to the flow direction. Since the effective cooling velocity to which an inclined sensor responds is the velocity component normal to the sensor, the anisotropy was measured with a hot-film sensor inclined at 45° to the flow. There is some sensitivity to the velocity component along the sensor axis; however, Champagne (1967) has suggested a method of correction. This was employed, but there is probably greater error in these measurements, perhaps $\pm 20\%$. The length to diameter ratio of the sensors used was 20, and Champagne's coefficient for the cooling effectiveness was determined to be $K = 0{\cdot}72$.

Our measurements of the turbulence anisotropy in each wake are compared in figure 7. Because the sense of the velocity gradient changes across the wake centre-line, the orientation of the sensor relative to the principal axis is reversed on opposite sides of the wake. Consequently, the same sensor responds to the maximum fluctuations on one side of the wake, and the minimum on the other. This is seen in the profiles. The principal stresses, $\overline{u_p^2}$ and $\overline{v_p^2}$, are shown rather than the turbulence intensities, u'_p and v'_p, because it is the anisotropy of the stresses that determines the effect of the turbulence on the mean flow. The index of anisotropy, $(\overline{u_p^2} - \overline{v_p^2})/(\overline{u_p^2} + \overline{v_p^2})$, is maximum in the shear layers at the perimeter of the near wake in both cases. However, the recirculation region behind the sphere mixes the turbulence and decreases the index to a minimum of 0·07 only two diameters downstream. Subsequent straining by the mean flow increases the anisotropy again, to a value of 0·36 at $x/d = 100$. In the wake of the disk, the index slowly decays to a minimum of 0·44 at $x/d = 30$. Beyond this point, the anisotropy increases at the same rate as observed in the wake of the sphere. However, the anisotropy is always greater, reaching 0·54 at $x/d = 100$. Thus the anisotropy continues to increase and the distribution of the principal stresses does not become self-preserving in either case.

Owing to the intermittency of the flow, the distribution of the principal stresses seen in figure 7 is somewhat misleading. The stresses are actually larger, and both principal stress components have relative maxima near the point where the mean strain rate is greatest. If it is assumed that the velocity fluctuations vanish in the irrotational flow outside the turbulent core, then the ordinary mean $\overline{u^2}$ is related to the turbulent zone

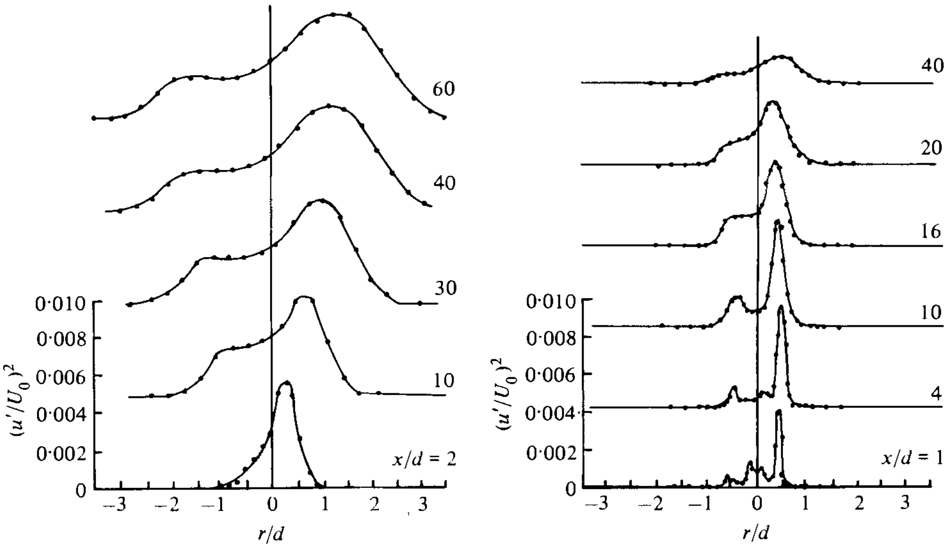


FIGURE 7. Profiles of the principal stresses in the wake of the sphere (left) and porous disk (right).

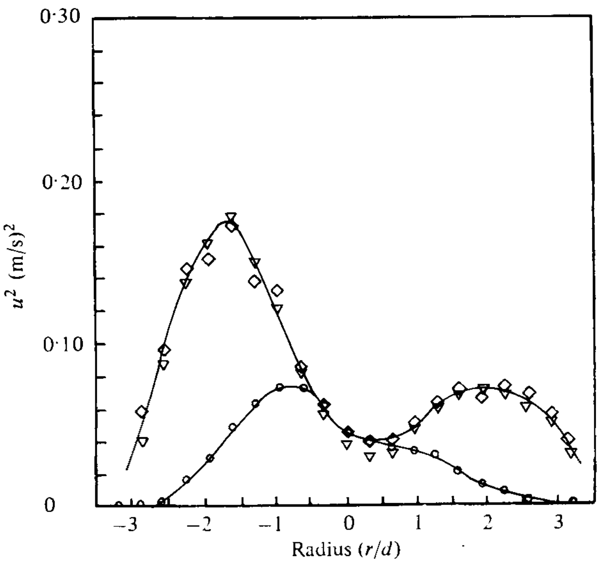


FIGURE 8. Turbulent zone averages of the principal stress components in the wake of the sphere. ○, conventional average; ◇, corrected for intermittency; ▽, measured zone average.

average $\overline{u_T^2}$ by the expression $\overline{u_T^2} = \overline{u^2}/\gamma$. The distributions of the principal stresses obtained by correcting for intermittency in this way are shown in figure 8, which is representative of profiles in the wake of the sphere. A peak value of the minimum stress becomes apparent. The zone average was also measured directly, using analog methods. The results of this procedure are also shown in the figure. Since the free-stream turbulence level is low, there is good agreement between methods.

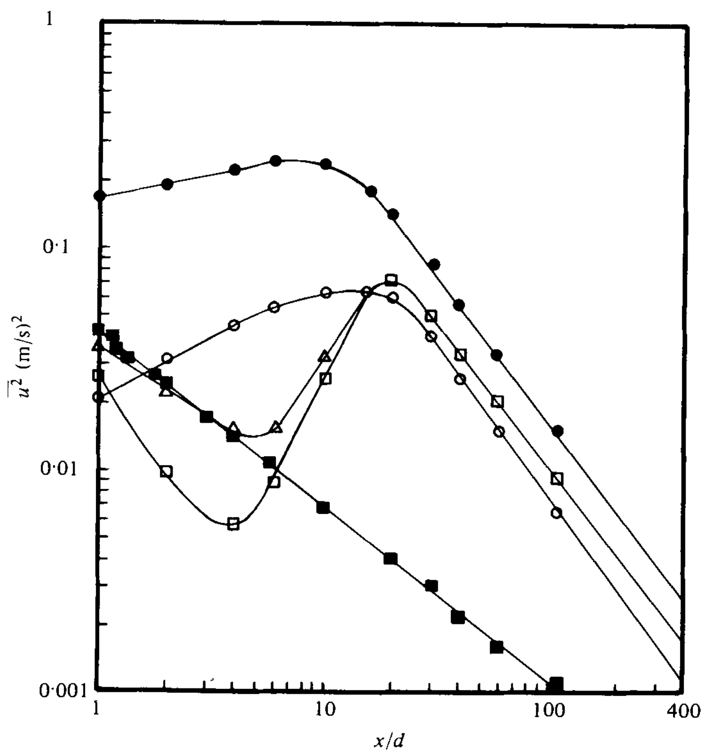


FIGURE 9. Axial variation of the mean-square turbulence intensity. ■, grid turbulence; □, $\overline{u^2}$ on the wake centre-line; △, $\overline{u_p^2}$ on the wake centre-line; ●, principal stress maxima; ○, principal stress minima.

The axial variation of the turbulent energy in the wake of the disk was also compared with the decay of the turbulence generated by a grid of the same porosity.† A similar screen was used to cover the entire cross-section of the tunnel. Since the mean flow is uniform, the grid turbulence is nearly isotropic (the index is 0·025) and should simply undergo viscous decay. On the other hand, the turbulence in the region immediately behind the disk is highly anisotropic (the index is 0·44 in the core and 0·80 in the shear layers) and is subject to straining by the mean velocity gradient, which amplifies the turbulent energy. Resulting differences in the decay of the turbulence are seen in figure 9. The initial intensity of the turbulence was observed to be the same in each case. The streamwise component of the turbulence, $\overline{u^2}$, behind the grid decays as $x^{-0 \cdot 81}$ over the entire range investigated. The turbulence in the core of the wake first goes through a stage of rapid decay in which $\overline{u^2}$ varies approximately as $x^{-1 \cdot 4}$. This region is about five diameters long. As the shear layer spreads to the axis, the turbulent energy in the core increases and the anisotropy at the centre of the wake approaches zero. The production of turbulence by the mean strain rate increases the turbulent energy during this stage, which is ten to fifteen diameters long. In the following stage of decay, the wake becomes self-preserving so that the production of turbulence is balanced by its dissipation and the turbulent energy decays at

† The authors wish to acknowledge the suggestion by Dr L. S. G. Kovasznay during a discussion of our preliminary results that such a comparison might be informative.

the same rate as a scalar contaminant, $\overline{u^2} \sim x^{-\frac{4}{3}}$. The final period of decay, in which molecular diffusion becomes dominant, was not attained within the range of this experiment.

4. Discussion of results

Differences in the initial structure of the turbulence have thus been seen to have a significant effect on the subsequent evolution of the round wake. The mean velocity and turbulence intensity profiles evolve the same self-preserving shapes in the wake of each model, and the development of the characteristic scales follows the same power laws in each case. However, the wake of the sphere spreads more rapidly than the wake of the porous disk, so that two different self-preserving flows have evolved. Since the surface porosity of the disk was chosen to ensure that both models had essentially the same drag, this difference is inconsistent with the equilibrium hypothesis that flows of the same 'type', with the same momentum integral, develop the same self-preserving state.

Because it will help to understand our conclusions, we shall review the meaning of self-preservation before discussing the present results. The assumption that the structure of a developing flow changes in scale but has a similar functional form at each stage of decay constitutes the hypothesis of self-preservation; Stewart & Townsend (1951) defined it as similarity during decay. For turbulent free shear flows self-preservation means that the shapes of the mean velocity defect and Reynolds stress profiles do not change with axial distance, so that they can be normalized with local scales of length and velocity, L_c and U_c. Thus we assume,

$$U = U_0 + U_c f(y/L_c) \quad \text{and} \quad -\overline{uv} = U_c^2 g(y/L_c), \tag{1}$$

in which U_0 is the uniform velocity at infinity and f and g are the profile functions. The physical constraints imposed by the conservation of momentum require that the scale parameters must be particular functions of distance from the origin. In general,

$$U_c = U_1(x/d)^{-m} \quad \text{and} \quad L_c = L_1(x/d)^n, \tag{2}$$

in which m and n are determined by the assumption of self-preservation, but U_1 and L_1 are empirical constants.

The form of the profile functions are also determined by the basic assumption. In self-preserving flows, f and g must be related (Tennekes & Lumley 1972), since the dominant scales of the turbulence must be proportional to the scales of the mean flow. Thus, the turbulent velocity fluctuations must be proportional to the characteristic mean flow velocity,

$$u \sim v \sim L_c \frac{\partial U}{\partial y}. \tag{3}$$

This is equivalent to saying that all velocities are proportional to U_c, but using this form of the proportionality for one of the turbulent velocities leads to the conclusion that

$$\overline{uv} \sim U_c L_c \frac{\partial U}{\partial y}. \tag{4}$$

In terms of the profile functions, this is

$$g = \frac{1}{R_T} f', \tag{5}$$

in which the turbulent Reynolds number R_T is an empirical constant of proportionality. The similarity solution for the shape of the velocity profile follows by substituting this relation into the appropriate equation of motion for each flow geometry, and then integrating.

By absorbing U_1 and L_1 into the value of R_T, only one empirical constant is required to specify the rate at which the flow develops. There is considerable variation in the measured values of R_T, but values of the order of 60 for wall flows, approximately 30 for jets, and about 15 for wakes and mixing layers are typical (Tennekes & Lumley 1972). The value of R_T depends on the relative size of the dominant eddy in each flow; that is, on the ratio l/L_c of the eddy scale to the scale of the flow. This result was derived by Townsend (1956) and Newman (Gartshore 1966) as a consequence of the energy equilibrium of the dominant eddy. It was verified by Gartshore (1966) for various simple shear flows, under the assumption that the dominant eddy scale is proportional to the amplitude of the motion of the turbulent interface. According to the equilibrium hypothesis, these differences in the characteristic value of R_T are due to differences in the total strain of each flow 'type'. That is, even though the vortex pair eddies are dominant in all fully developed flows, characteristic differences in the total strain of the turbulence cause differences in the scale of the eddy and, as a result, in the value of R_T. A more complete discussion of these points may be found in Townsend (1976).

Strictly then, the assumption of self-preservation is that the developing velocity and stress profiles can be normalized by the same scales of length and velocity. As a consequence of this assumption, there are characteristic power laws for the development of the scales and particular solutions for the shapes of the profiles. However, it is important to recognize that, while the turbulence is dominated by eddies of the order of the characteristic scales, neither the origin and structure of these eddies nor the variation in the values of the turbulent Reynolds number are determined by the assumption of self-preservation. These are currently understood in terms of a vortex stretching mechanism, according to the equilibrium hypothesis (Tennekes & Lumley 1972; Reynolds 1974).

From this discussion, it should be clear that our experimental results cannot be explained by a hypothesis that requires the same self-preserving state to develop in two wakes having the same momentum integral. We therefore suggest the following eddy structure hypothesis to explain the appearance of a different self-preserving wake behind each model. The self-preserving forms of the mean velocity and Reynolds stress profiles evolve because the turbulence is dominated by eddies which provide the particular connexion between the turbulence and the mean flow required by (3). In the wake of the porous disk, this connexion probably results because the vortex pair eddies become dominant. Since no large eddies are initially shed into this wake, the pair eddies would arise from straining of the turbulence by the mean flow. In this case the connexion necessary for self-preservation results because the strain rate of the pair eddies (u/l) must be proportional to the mean strain rate $(\partial U/\partial y)$. The

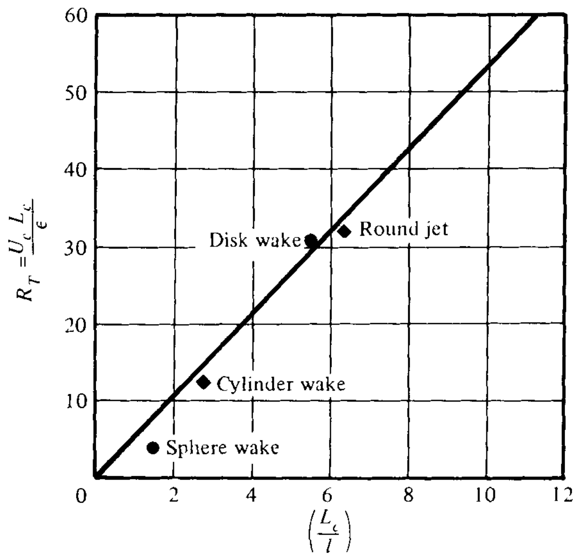


FIGURE 10. Turbulent Reynolds number R_T versus the relative eddy scale.

development of the flow is thus controlled by the vortex stretching mechanism of the equilibrium hypothesis.

On the other hand, large vortices are shed into the wake of the sphere; in the visualization experiment (figure 4) these were observed to persist for several hundred diameters. This slow fading of the turbulence memory means that the transverse vortices, not the vortex pair eddies, dominate the wake of the sphere. In this case the necessary connexion results because the vorticity of the large eddies (u/l) is proportional to the vorticity of the mean flow ($\partial U/\partial y$). But here development of the flow is controlled by the persistence of the transverse vortices. Our hypothesis is that the difference in the value of R_T is due to this difference in the mechanism of self-preservation.

This explanation of the wake differences has implications regarding the value of R_T in other flows. From the data in figure 10, R_T is 4 in the wake of the sphere and 31 in the wake of the porous disk. As shown in figure 10, these values are correlated with the eddy scale according to the relation developed by Gartshore (1966). Although the value for the sphere is representative of wake flows, the value for the porous disk is in the range typical of jet flows. Since jets also originate without vortex shedding, the value of R_T may actually depend on the structure of the dominant eddy. Thus, for wakes and mixing layers, in which the transverse vortices have been observed, the value of R_T is approximately 10. For jets and flows in which the vortex pair eddies become dominant, the value is about 30. Even though simple wall jets and boundary layers do not have self-preserving stages of development, the horseshoe vortices observed in these flows have an effective value of R_T near 60. Such a classification is probably too simple, but the apparent correlation between R_T and eddy type suggests that the effect of the large eddies should be modelled by changing the 'constants' in the turbulence equation according to the dominant eddy type, rather than the 'type' of flow.

Some additional comments regarding self-preservation may be appropriate. From observations of a perturbed wake, Narasimha & Prabhu (1972) proposed that a distinction between self-preserving and equilibrium flows be made at the second moment of the velocity fluctuations. That is, they proposed that a flow be identified as self-preserving if only the mean velocity profiles exhibit similarity, but that a flow is in equilibrium if the turbulent stresses are similar also. However, the wake of the sphere cannot be in equilibrium. Even though the effect of the large eddies in this wake is to make both the mean velocity and the turbulent stress profiles self-preserving within experimental error, the persistence of these eddies means that the structure of the turbulence is still far from being independent of initial conditions. This observation suggests that there are varying degrees of self-preservation.

For this reason we find it appropriate to introduce the concept of a self-preservation hierarchy. We define self-preservation of order one: the state when the mean velocity profiles are self-preserving; of order two: when in addition the Reynolds stress profiles are self-preserving, and so on through the higher-order moments. An equilibrium, or fully developed, flow would then be the asymptotic state that occurs when all the moments are self-preserving.

It is not clear, however, that anything can be inferred about the asymptotic state from measurements made in the self-preserving zone still influenced by memory effects. Grant's (1958) correlation measurements show that the vortex pair eddies have appeared by $x/d = 500$ in the wake of a cylinder. The appearance of this new vortex structure may be associated with a self-preserving state unrelated to the initial one. In fact, we can speculate that if the vortex pair eddies do eventually appear in all free shear flows, then all these flows will tend toward the same equilibrium state, characterized by a value of R_T near 30. The nature of the transition from one eddy type to the other is not known. Grant (1958) suggested that random perturbations of the transverse vortices may be stretched out in a loop to form the pair eddies, while Konrad (1976) suggested that they develop from a Taylor instability of the flow around the transverse vortices. However, the basic vortex stretching process remains a likely mechanism, because there is a region of large strain rate between the transverse vortices. Clearly, the present study raises many intriguing questions.

5. Summary

Measurements have been made in the turbulent wakes of a polished sphere and a porous disk having the same drag. Large vortices are periodically shed into the wake of the sphere from a toroidal eddy which is attached to the rear of the model. As a result, the wake boundary becomes intermittent just behind the sphere and the fully turbulent core of the wake is only about fifteen diameters long. On the other hand, enough flow comes through the porous disk to prevent the formation of a recirculating eddy. Since there is no vortex shedding, intermittency is not observed until approximately twenty diameters downstream of the disk and the fully turbulent core is more than one hundred diameters long.

In our experiments we found that both velocity and Reynolds-stress profiles reached the self-preserving state within a few diameters of the sphere and the porous disk. However, we found that the two states were different notwithstanding that both flows had the same momentum defficiency, and contrary to the implication of Townsend's equilibrium hypothesis that two bodies having the same drag will have the same self-preserving wake. Our experiments extended only to about 100 diameters downstream. We found that the large vortices shed behind the sphere still persist there. It is, in fact, these eddies that dominate the dynamics of the turbulence and produce a state of self-preservation in the velocity and Reynolds stresses. On the other hand because of the persistence of these eddies the higher-order moments of the turbulence cannot be expected to be self-preserving. In our work we introduced the concept of a hierarchy of self-preservation. We defined as self-preservation of order one, the state when the mean velocity profiles are self-preserving; of order two, when in addition the Reynolds stresses are self-preserving and so on through the higher-order moments. From this point of view the self-preserving region of the sphere, where the large eddies are dominant, is of order two and is characterized by a value of the flow constant near $R_T \sim 4$. Even though we did not conduct experiments beyond 100 diameters downstream we suspect that in the case of the sphere, if and when the large coherent eddies fade and the vortex pairs appear, a sequence of self-preserving profiles of higher and higher order will be formed as we proceed downstream reaching asymptotically an equilibrium state characterized perhaps by a universal value of the flow constant R_T. The wake of the porous disk took longer to become self-preserving, and when it did so, it reached the value of the flow constant $R_T \sim 30$, a value usually associated with jets.

The present work has shown that all self-preserving flows are not necessarily equilibrium flows but of course flows which are in equilibrium must also be self-preserving. Furthermore, if observed differences in the self-preserving state of the various flow geometries are actually due to differences in the initial conditions, then all shear flows (wakes, jets, mixing layers, etc.) may in fact tend asymptotically toward the same equilibrium state, characterized by the appearance of the vortex pair eddies and a universal value of the flow constant $R_T = 30$. Thus, the present results serve to narrow the meaning of equilibrium, while emphasizing that self-preserving flows can still depend on initial conditions.

The authors wish to acknowledge with gratitude the financial support of the National Science Foundation under Grant GK-23694.

REFERENCES

ACHENBACH, E. 1974 Vortex shedding from spheres. *J. Fluid Mech.* **62**, 209–221.

ACHENBACH, E. 1974 Effects of surface roughness and tunnel blockage on the flow past spheres. *J. Fluid Mech.* **65**, 113–125.

BALDWIN, L. V. & SANDBORN, V. A. 1968 Intermittency of far wake turbulence. *A.I.A.A. J.* **6**, 1163–1164.

BEVILAQUA, P. M. 1973 Intermittency, the entrainment problem. Ph.D. thesis, Purdue University.

BEVILAQUA, P. M. & LYKOUDIS, P. S. 1971 Mechanism of entrainment in turbulent wakes. *A.I.A.A. J.* **9**, 1657–1659.

BROWN, G. L. & ROSHKO, A. 1971 Effect of density differences on the turbulent mixing layer. *Turbulent Shear Flows. AGARD Conf. Proc.* no. 93, paper 23, pp. 1–12.

CARMODY, T. 1963 Establishment of the wake behind a disk. Ph.D. thesis, State University of Iowa.

CASTRO, I. P. 1971 Wake characteristics of two-dimensional perforated plates normal to an air stream. *J. Fluid Mech.* **46**, 599–609.

CHAMPAGNE, B. H. 1967 Turbulence measurements with inclined hot wires. *J. Fluid Mech.* **28**, 177–182.

FREMUTH, P. & UBEROI, M. S. 1973 Temperature fluctuations in the turbulent wake behind an optically heated sphere. *Phys. Fluids* **16**, 161–168.

GARTSHORE, I. S. 1966 Experimental examination of the large eddy equilibrium hypothesis. *J. Fluid Mech.* **24**, 89–98.

GIBSON, C. H., CHEN, C. C. & LIN, S. C. 1968 Measurements of turbulent velocity and temperature fluctuations in the wake of a sphere. *A.I.A.A. J.* **6**, 642–649.

GRANT, H. L. 1958 Large eddies of turbulent motion. *J. Fluid Mech.* **4**, 149–190.

HWANG, N. H. & BALDWIN, L. V. 1966 Decay of turbulence in axisymmetric wakes. *A.S.M.E. J. Basic Engng*, March, pp. 261–268.

KIBENS, V. R. 1968 Intermittent region of a turbulent boundary layer. Ph.D. thesis, Johns Hopkins University.

KLINE, S. J., REYNOLDS, W. C., SCHRAUB, F. A. & RUNSTADLER, P. W. 1967 Structure of turbulent boundary layers. *J. Fluid Mech.* **30**, 741–773.

KONRAD, J. H. 1976 Experimental investigation of mixing in shear layers and wakes. Ph.D. thesis, California Institute of Technology, Pasadena, California.

NARASIMHA, R. & PRABHU, A. 1972 Equilibrium and relaxation in wakes. *J. Fluid Mech.* **54**, 1–17.

OSWALD, L. J. & KIBENS, V. R. 1971 Turbulent flow in the wake of a disk. *Univ. Michigan Tech. Rep.* no. 002820.

PAO, H. P. & KAO, T. W. 1977 Vortex structure in the wake of a sphere. *Phys. Fluids* **20**, 187–191.

PAPAILIOU, D. D. 1971 Turbulent vortex streets. Ph.D. thesis, Purdue University.

PAPAILIOU, D. D. & LYKOUDIS, P. S. 1974 Turbulent vortex streets and the mechanism of entrainment. *J. Fluid Mech.* **62**, 11–31.

REYNOLDS, A. J. 1974 *Turbulent Flows in Engineering*. Wiley.

RIDDHAGNI, P. R., BEVILAQUA, P. M. & LYKOUDIS, P. S. 1971 Measurements in the turbulent wake of a sphere. *A.I.A.A. J.* **9**, 1433–1434.

STEWART, R. W. & TOWNSEND, A. A. 1951 Similarity and self-preservation in isotropic turbulence. *Phil. Trans. Roy. Soc.* A **243**, 359–386.

TENNEKES, H. & LUMLEY, J. L. 1972 *First Course in Turbulence*. MIT Press.

TOWNSEND, A. A. 1949 Fully developed turbulent wake of a circular cylinder. *Australian J. Sci. Res.* **2**, 451–468.

TOWNSEND, A. A. 1956 *Structure of Turbulent Shear Flow*. Cambridge University Press.

TOWNSEND, A. A. 1970 Entrainment and the structure of turbulent flow. *J. Fluid Mech.* **41**, 13–46.

TOWNSEND, A. A. 1976 *Structure of Turbulent Shear Flow*, 2nd edn. Cambridge University Press.

UBEROI, M. S. & FREYMUTH, P. 1970 Turbulent energy balance and spectra of the axisymmetric wake. *Phys. Fluids* **13**, 2205–2210.